ARMY RESEARCH LABORATORY

Patterned Armor Performance Evaluation for Multiple Impacts

by William S. de Rosset

ARL-TR-3038 August 2003

Army Research Laboratory
Aberdeen Proving Ground, MD 21005-5069

ARL-TR-3038 **August 2003**

Patterned Armor Performance Evaluation for Multiple Impacts

William S. de Rosset
Weapons and Materials Research Directorate, ARL

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED (From - To)
August 2003	Final	June 2002–June 2003

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Patterned Armor Performance Evaluation for Multiple Impacts	
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
William S. de Rosset	AH80
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
U.S. Army Research Laboratory ATTN: AMSRL-WM-MD Aberdeen Proving Ground, MD 21005-5069	ARL-TR-3038
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Patterned armor is characterized by an array of repeating cells, such as tiled ceramic armor or reactive armor boxes. Performance characteristics of an ideal patterned armor with respect to multiple hits are discussed, and the types of single-shot ballistic data needed to quantify that performance are presented. An approach to use these data is developed to provide a quantitative measure (probability of nonperforation after a given number of impacts) of the patterned armor performance against multiple impacts. This performance measure can then be compared to a well-posed multiple-hit criterion to assess whether the patterned armor meets the criterion.

15. SUBJECT TERMS

patterned armor, armor performance evaluation, limit velocity, ceramic armor, multiple-hit criterion, lightweight armor, multiple-hit capability

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					William S. de Rosset
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (*Include area code*)
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL	28	(410) 306-0816

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

Contents

List of Figures

Acknowledgments

The author is indebted to Dr. Steven Segletes for his careful review of the draft manuscript and his general formulation of the probability of nonperforation of patterned armor containing cells capable of withstanding more than one impact, which is used as the Appendix to this report. Dr. Segletes provided the generalization to the combinatorial problem used concerning multiple hits on a given cell.

Dr. Segletes would like to thank Dr. Paul Tanenbaum of the U.S. Army Research Laboratory's Survivability/Lethality Analysis Directorate for discussing this problem at length and making very helpful suggestions, including that an inclusion/exclusion approach was applicable to a simpler, though similar, problem and should apply here as well. Dr. Tanenbaum's review of the Appendix further corrected some deficient definitions and notations.

INTENTIONALLY LEFT BLANK.

1. Introduction

At the 14th Annual Ground Vehicle Survivability Symposium in Monterey, CA, on April 2003, LTG John S. Caldwell called for armors to be designed that would be perforated 0% of the time. This is certainly the goal of all armor designers, but, in practice, certain compromises must be made between protection levels, cost, and vehicle weight such that this goal is rarely attained. This is especially true for lightweight, air-transportable fighting vehicles that the Army is currently developing as part of the Future Combat System of Systems.

The protection problem for lightweight armored vehicles is compounded by threats that feature multiple impacts, such as machine guns and fragmenting warheads. Armors designed for these types of vehicles must possess the capability to defeat a large number of closely spaced impacts in a given burst or explosion. Some of the armors being considered for lightweight vehicles are modular in nature, have a repeating geometric pattern, or both. These will be referred to as patterned armor. The module or repeating pattern element will be referred to as a cell.

While specific cases of patterned armor performance have been examined in the past, there has been no general approach developed to evaluate patterned armor for lightweight armored vehicles attacked by automatic weapons or bursting munitions. Whatever level of performance is set as the multiple-hit criterion, there has to be an accepted means or methodology by which the results of single-shot ballistic data can be used to estimate the performance of a given patterned armor design against multiple impacts.

Section 2 will contain a discussion of a possible testing strategy and the considerations that must be made in conducting the ballistic tests. Characteristics of effective patterned armor will be included in this section.

Section 3 will provide a proposed methodology that shows how the individual ballistic data might be used to establish the performance level of the patterned armor against multiple impacts. It is based on a model recently published by de Rosset and Wald (2002). Several examples will be presented that exercise this methodology and show its usefulness in evaluating trade-offs in a given armor technology, trade-offs between competing armor technologies, and effects of velocity on armor performance. The final section will summarize the main ideas presented in the report.

2. Patterned Armor Performance Characteristics

For the sake of clarity and purposes of discussion in this report, it is necessary to distinguish between the terms "ballistic performance" and "multiple-hit criterion." Ballistic performance of

a patterned armor vs. multiple impacts can be determined from an analysis of individual ballistic results and is the subject of this report. Multiple-hit criterion is a specified level of performance on a given armor cell or armor cell array. This level of performance should be set high enough to allow the armored vehicle to survive most realistic battlefield encounters. For instance, the multiple-hit criterion may state that the armor array must withstand perforation of a 10-round burst from a 12.7-mm machine gun at a distance of 500 m, with a probability of 90%. The weapon-to-target distance (range) and the weapon characteristics give the dispersion of the impacts on the target as well as the striking velocity.

The procedures used to evaluate patterned armor that are presented in this section are straightforward and based on the nature of the armor being tested. Most, if not all, of these procedures have been and are being used by the U.S. Army Research Laboratory in testing ceramic tiled armor arrays. These procedures are to be distinguished from such tests as depth-of penetration (DOP) tests or other types of screening tests that are used in the early stages of evaluating armor concepts and materials. The procedures presented here are to be used for armor arrays that have already undergone some initial tests and have high potential for actual fielding on an armored vehicle.

The main characteristic of patterned armor is that the armor is made up of repeating cells. There is generally a border or line of demarcation between the cells that is a potential weak point in the armor array. For instance, the seam between ceramic tiles generally has lower ballistic performance than the center of the tile. It may also be that performance of an individual cell depends on the hit location within the cell. Reactive armor is a good example of this (de Rosset, 1998). To ensure overall armor array performance, an impact in one cell should leave adjoining cells intact and capable of defeating a subsequent impact (if it had not been previously struck). The properties of an ideal patterned armor can thus be summarized as follows:

- ability of an undamaged cell to defeat a single threat round at a velocity corresponding to the specified range (based on limit velocity as indicated next);
- uniform ballistic performance over the entire area of the cell;
- no loss of performance in the area between cells (seam effect); and
- no effect on adjoining cells from impact on a given cell.

The first step in the evaluation procedure is to determine the limit velocity of the threat against an individual cell. This has usually been accomplished though an initial screening process that has determined suitable materials and design. By suitable, we mean that the armor material and design produces a limit velocity that is high enough to meet the multiple-hit criterion for a single shot (rather than the entire burst). The limit velocity can be determined in either of two ways. First, a plot of the residual velocity vs. striking velocity will yield a limit velocity V_L through

a Lambert-Jonas (Lambert and Jonas, 1976) fit to the data. Second, a V_{50} can be determined through standard test procedures. Where there is little or no zone of mixed results, the two values are approximately equal. For purposes of discussion, this report will assume that there is a single value for the limit velocity. Above this velocity, the armor is perforated by the threat, and below this value, the threat is defeated. This simplistic approach can be modified to take into account a more sophisticated concept of probability of perforation. However, the essence of the methodology presented in this report would not be affected with this more involved approach.

The reduction in striking velocity as a function of range (weapon to target distance) is rather steep for small arms ammunition. It is conceivable that there is a range at which a given tile can withstand two impacts due to the low-striking velocity. The range where two impacts can be sustained by a single tile might vary slightly as a function of the distance between impacts on that tile. It is expected that with new ceramic tile encapsulation technology, the probability of a single tile sustaining multiple hits will increase. The next section indicates how the evaluation methodology can be adapted to take into account this increase in performance.

The next step is to determine the uniformity (or lack thereof) of the ballistic performance over the entire area of the cell. Ideally, the limit velocity is the same at all impact points within the cell. However, if there is some falloff in performance in some particular area of the cell, a performance map of the cell needs to be generated as input data for the evaluation methodology. Clearly, it is more advantageous to apply efforts to making the performance homogeneous over the entire cell than to expend resources to show where the cell fails to meet the requirements. However, there may simply be physical reasons why the performance cannot be uniform.

The third step is to determine the performance of the patterned armor in the area between cells. For these tests, multiple cells must be employed in an array that simulates to the greatest extent practical the actual arrangement of the cells in the armor design. Limit velocities should be determined not only for straight-edge seams but also for individual points where three cells might meet (triple point).

Finally, ballistic tests must demonstrate that an impact on one cell does not affect the performance of an adjacent cell upon subsequent impact. This is done by shooting at a target with an array of cells put together similar to what is envisioned for the fielded armor design. Impacts on adjacent cells should be defeated consistently at the velocity specified in the multiple-hit criterion.

Clearly, it may not be possible to design a patterned armor with all the ideal characteristics noted, especially when there are limits to the allowable areal density and cost. However, a sufficient amount of ballistic data must be gathered on the armor designs under consideration so that the trade-offs that may be required can be quantified to the greatest extent possible. The next section presents a methodology that can be used in this type of study.

3. Methodology for Assessing Patterned Armor Performance Against Multiple Hits

3.1 Basic Methodology

The basic methodology for assessing patterned armor performance has already been presented in a previous report (de Rosset and Wald, 2002). The basic concepts for ceramic armor are readily transferable to patterned armor in general. The methodology is based on the calculation of the probability of perforating a given patterned armor array with one or more impacts. The dispersion of the impacts on the armor array is assumed to be in a bivariate normal distribution and is related to the range-dispersion characteristics of the weapon firing the ammunition. The report also shows that using a random distribution of impacts on the armor array gives similar results, with the largest differences in the calculations occurring for a small number of impacts (less than five) at short ranges (less than 250 m). For simplicity, this report will use a random distribution of impacts on the armor array.

Several assumptions go into this methodology. First, it is assumed that all the relevant ballistic data have been obtained. In particular, any nonuniformity in ballistic performance over the cell face has been determined. Second, it is assumed that the armor has been designed such that impact on one cell will not affect the performance of adjacent cells. This would appear to be a reasonable requirement on a viable patterned armor. Third, it is assumed that when a cell is impacted, it is no longer capable of defeating a subsequent impact by the threat round. (This assumption will be modified in a variation of this methodology later in the report.) We also take a square with side length D as the cell geometry, although other shapes can be used in the analysis (hexagons, triangles, and rectangles, for instance). The area A (assumed to be a circle) over which the impacts fall on the target can be related to the range-dispersion characteristics of the attacking weapon.

Consider the arrangement of square cells as shown in Figure 1. Each of the cells has uniform ballistic performance and can defeat the threat round at the specified velocity (i.e., the limit velocity is above the threat velocity). For computational simplicity, the bullet diameter is taken as 0. Thus, it cannot impact two cells at one time. The first round that impacts the target has a 100% probability of being defeated (i.e., no perforation). The second round will have a finite probability of perforating the target since it may land on the first tile struck. If N tiles cover the area A ($A = N*D^2$, neglecting corner effects), the probability P_2 that the second round is defeated is given by

$$P_2 = P_1*(A - D^2)/A = P_1*(1 - D^2/A), \tag{1}$$

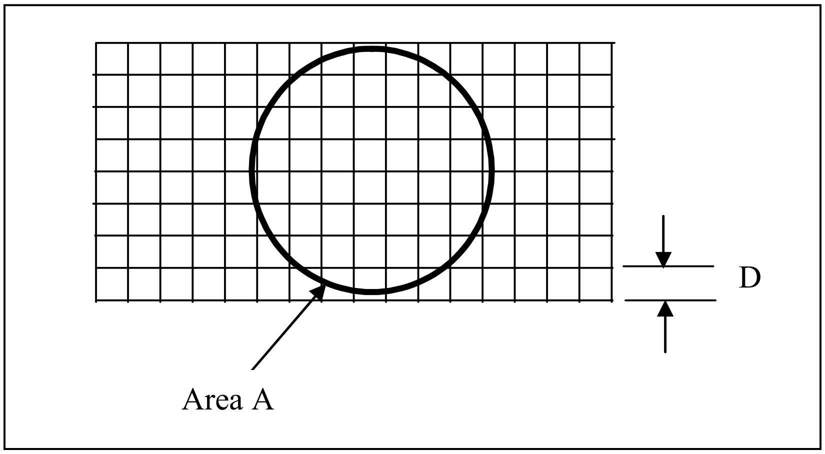

Figure 1. Patterned armor cell array showing D and A.

where $P_1 = 1$. As subsequent cells are struck and become ineffective in stopping the threat rounds, the probability P(n) of the target not being perforated after n rounds have impacted it is given by

$$P(n) = (1 - D^2/A)*(1 - 2*D^2/A)*(1 - 3*D^2/A)*\ldots*(1 - (n - 1)*D^2/A). \quad (2)$$

De Rosset and Wald (2002) also showed that if the cell contained a zone at the edge of the cell that could not stop the threat round, the probability P(n) of stopping n shots was given by

$$P(n) = (1 - D^2/A)*(1 - 2*D^2/A)*\ldots*(1 - (n - 1)*D^2/A)*(1 - 2*\delta/D)^{2n}, \quad (3)$$

where δ is the width of the area of low performance. Note that P(n) is not defined as the probability that the n^{th} shot is defeated.

3.2 Patterned Armor Performance Comparison for Similar Designs

Equations 2 and 3 can be used to determine which of two similar patterned armor designs is better. Suppose, for instance, that the armor designer is able to produce a patterned armor with a cell of side length D that has constant ballistic performance across its face and stops the threat at the specified velocity. It was shown previously (de Rosset and Wald, 2002) that the performance of a ceramic tile array could be increased with a decreasing cell size. What if another design is produced that has a smaller cell side length but has a small zone of low performance near the edges? Which design is better will obviously depend on the actual ballistic data. For demonstration purposes, several input parameters will be chosen arbitrarily and fixed to demonstrate how the methodology can be applied.

Take $A = 2m^2$ (corresponding to a dispersion of about 500 mm), the large cell edge length as 100 mm, and the small cell edge length as 90 mm. Figure 2 shows the results for a value of 1 mm for δ, the width of the low-performance zone.

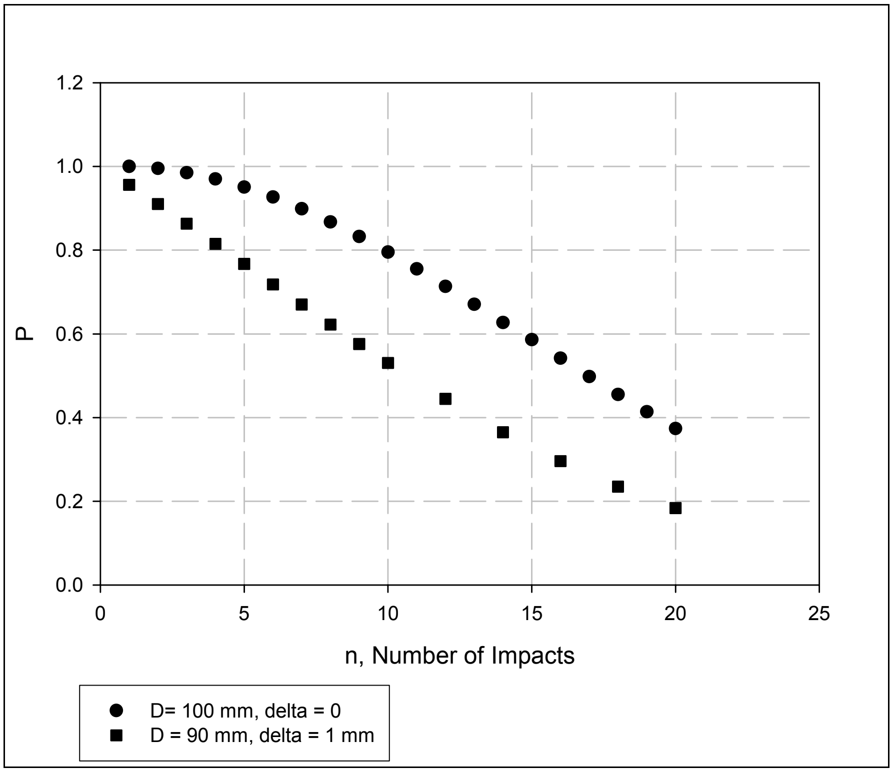

Figure 2. Probability of nonperforation P(n) given n impacts for two cell types.

The methodology shows the value of having no zone of weak performance. The calculations continue with smaller cell sizes but the same value of delta (δ). The difference in armor performance grows due to the fact that as the number of cells increases in the impact zone (area A), the negative effect of increasing vulnerable area overwhelms the positive effect of having a smaller cell size. The methodology can easily be applied to more complicated situations where delta is nonzero and varies with cell size.

3.3 Patterned Armor Performance Comparison for Different Armor Technologies

Consider now a patterned armor with the following properties. First, it has a very small cell size, ~3 actual bullet diameters on a side. (Of course, the calculations still use a bullet with 0 diameter.) Second, ballistic tests show this particular armor can defeat the threat bullet 19 out of 20 times. That is, the threat is able to perforate the target 1 out of 20 times, even when impacting an undamaged cell. Third, there is no zone of low performance ($\delta = 0$). These characteristics are purely hypothetical, and the example is offered to show how the proposed methodology might be used to evaluate a patterned armor where performance across the cell face may vary in a way that is different from that generally associated with ceramic tiles. The armor concept will be referred to as B4600 Armor.

The methodology is modified in the following way. The cell performance is modeled by assuming that there is a 5% probability that any impact on an intact (i.e., undamaged) cell will

perforate the cell. We use a square unit cell and proceed as before. If the impact area corresponding to a given dispersion is A, then the probability P_1 that the armor stops the first shot is

$$P_1 = 0.95*N*D^2/(N*D^2) = 0.95, \tag{4}$$

where N (an integer) is the number of cells contained in A and D is the length of the edge of the unit cell. As before, $N*D^2$ is taken as equal to A (a good approximation for large A and small D).

The second round will see a target with one unit cell missing, as well as a 5% probability of perforating each of the remaining N – 1 unit cells. Thus, the probability of stopping the second impact will be given by

$$P_2 = (0.95*(N-1)*D^2)/(N*D^2). \tag{5}$$

For n shots, the probability P of defeating them all is then

$$P(n) = P_1*P_2*P_3*\ldots*P_n$$

$$= 0.95*N*0.95*(N-1)*0.95*(N-2)*\ldots 0.95*(N-n+1)*D^{2n}/(N*D^2)^n\ , \tag{6}$$

or

$$P(n) = 0.95^n*N*(N-1)*(N-2)*\ldots*(N-n+1)/N^n\ . \tag{7}$$

The B4600 Armor can be compared to a traditional ceramic tile array using equations 3 and 7. As in the previous example, choice of input parameters is arbitrary, and the comparison is made for demonstration purposes only. For the B4600 Armor, a cell edge length of 25 mm is selected. Using an area of 2 m^2 gives N = 3200. Figure 3 compares the performance of B4600 Armor to that of a patterned armor, with a square cell 90 mm on a side and a delta of 1 mm.

For these particular input parameters, the B4600 Armor has a slightly lower performance for $n < 5$ but then outperforms the ceramic tile armor for $n > 5$. Different results would be obtained with different input parameters.

3.4 Velocity Effects

Based on the dispersion characteristics of the threat weapon, the methodology presented to this point takes into account the effect of weapon-to-target distance (range) through an increase in impacted area. That is, as the range is increased, A increases and the probability of no

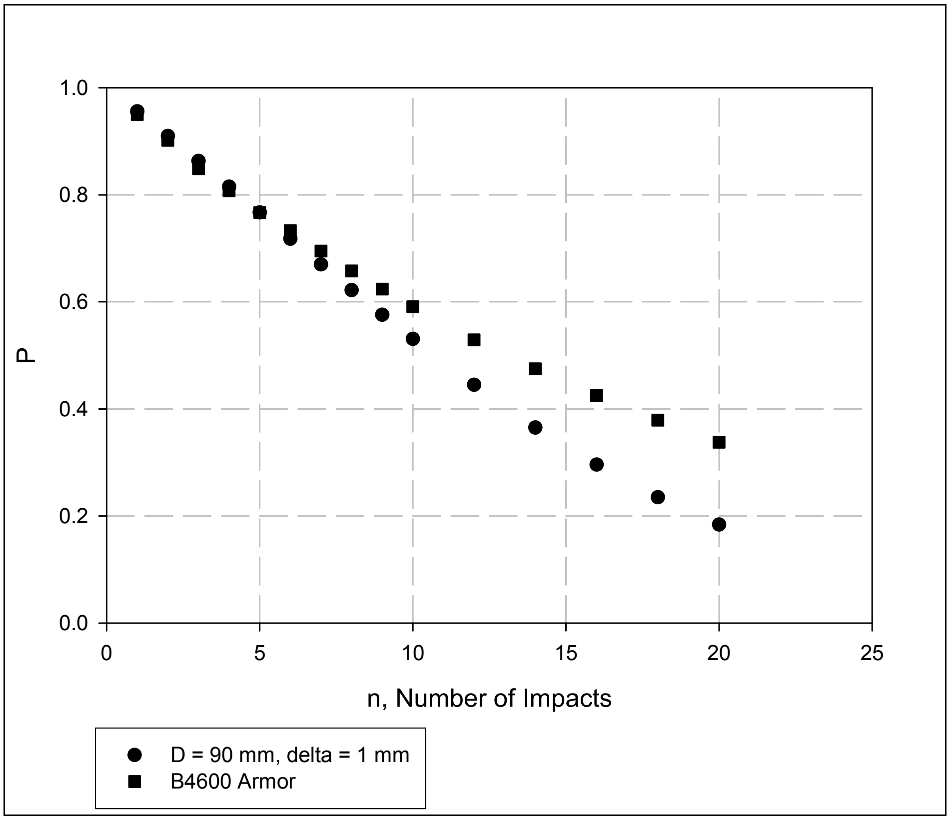

Figure 3. Performance comparison between tiled ceramic and B4600 Armor.

perforations for a given number of impacts goes up. However, an increase in range also means that the impact velocity of a given round decreases. For small- and medium-caliber weapons, this decrease can be substantial. For instance, the impact velocity as a function of range is shown in Figure 4 for the 12.7-mm AP-M2 machine gun round (Mascianica, 1976). The large reduction in velocity has not been taken into account up to now. However, the examples given show that it is quite easy to incorporate velocity effects into the methodology, provided that the ballistic data exists.

A trivially obvious consideration is the range at which the impact velocity equals the limit velocity of a given cell. For ranges less than this value, perforations will occur with all impacts. For ranges greater than this value, the methodology presented here can be employed. Clearly, a goal of the armor designer is to produce a cell that has a limit velocity above the muzzle velocity of the threat weapon.

Next, consider the case where the limit velocity of a given cell against a specified threat is higher than the weapon's muzzle velocity. Suppose, however, there is a zone of low performance that is dependent on velocity. At some impact velocity, the round can be defeated in the zone of low performance. The armor performance thus increases with increasing range for two reasons—

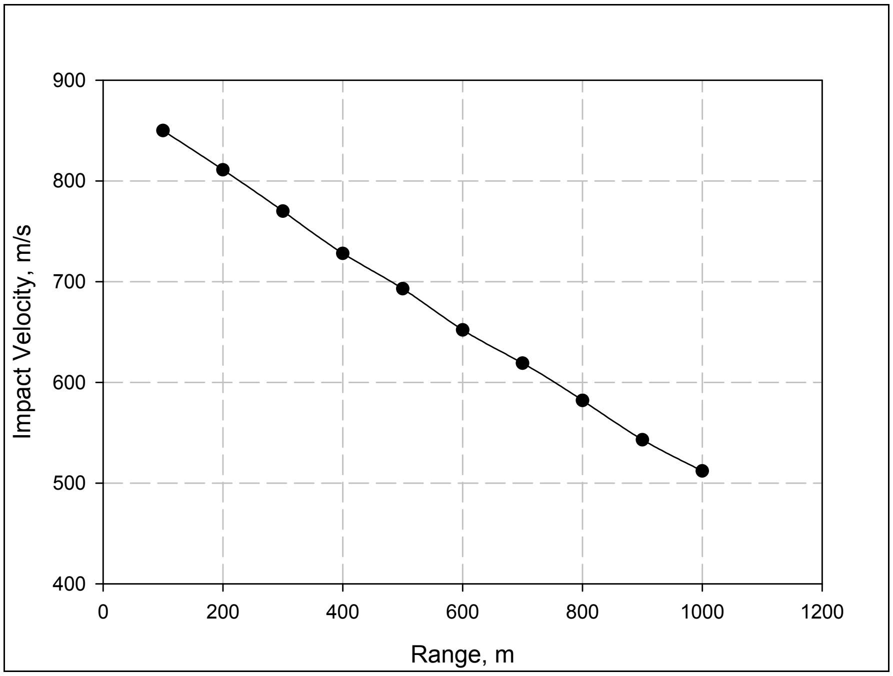

Figure 4. Impact velocity vs. range for the 12.7-mm AP-M2 machine gun round.

increase in A and an increase in cell area capable of defeating the round. The second reason will result in an incremental increase in P at the range corresponding to the velocity where the zone of weak performance is eliminated.

Finally, it is a reasonable assumption that at some impact velocity, a given cell will be able to defeat more than one impact. It may not happen in all cases, especially if the hit locations on a given cell are very close. In addition, the assumption might not hold for reactive armor. For the sake of argument, however, assume that at some velocity, a cell can defeat two impacts. At the range corresponding to this velocity, there will be another incremental increase in P, the probability that the armor array is not perforated after some number of impacts.

To calculate this probability, consider an array of N cells representing a patterned armor. Let $P(m)$ be the probability that all shots up to and including the m^{th} shot are stopped by the array. If $P_A(m)$ is the probability of the m^{th} shot hitting a cell that has not been struck before in an array that has survived without perforation and $P_B(m)$ is the probability of the m^{th} shot hitting a cell that has been hit just once in an array that has not been perforated, then

$$P(m) = P_A(m) + P_B(m). \tag{8}$$

Now let $Q_A(m-1)$ equal the sum of all undamaged tiles for all possible variations of target configurations that have survived up to (but not including) shot number m. In a similar manner, $Q_B(m-1)$ is the sum of all single-impacted tiles in arrays that have survived up to (but not including) shot number m. The number of cells in all combinations of outcomes through the m^{th} shot is N^m. Thus,

$$P_A(m) = Q_A(m - 1)/N^m \text{, and } P_B(m) = Q_B(m - 1)/N^m . \quad (9)$$

To delineate the possible combinations of outcomes from the shots on the patterned array, the following notation will be employed. Let $(x_0, x_1, x_2, x_3,\ldots)$ represent an array that has x_0 cells with no impacts, x_1 cells with one impact, x_2 cells with two impacts, and so forth. Clearly,

$$\Sigma\, ix_i = m,$$

and (10)

$$\Sigma\, x_i = N,$$

where it goes from 0 to the number equal to the largest number of impacts in a cell in the array. There will be many combinations for a given set of x_i, and the frequency of each combination may also be different. The primary interest is in the number of cells with no impacts ($x = x_0$) or just a single impact ($x = x_1$). Note also that x_i is either 0 or a positive integer and cannot exceed m.

As an example, take N = 3. Before any impacts occur, $Q_A(0) = 3$ and $Q_B(0) = 0$. In general, before there are any impacts, the only possible arrangement is (N), and

$$Q_A(0) = N, \text{ and } Q_B(0) = 0. \quad (11)$$

The probability of nonperforation resulting from the first shot for our specific example is given by

$$P(1) = 3/3^1 + 0/3^1 = 1. \quad (12)$$

In general,

$$P(1) = N/N^1 + 0/N^1 = 1. \quad (13)$$

The possible target arrangements for the N = 3 array after the first impact are shown in Figure 5. A cell that is impacted is indicated by O.

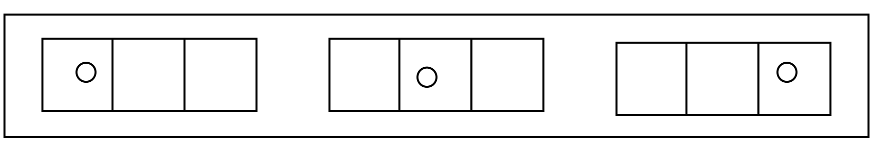

Figure 5. Possible outcomes from the first impact for N = 3.

This shot results in an array designated by (2, 1). That is, there will be two undamaged cells and one damaged cell for each possible array. In general, the resulting configuration will be designated as (N – 1, 1). For the specific case, the total number possible of undamaged cells is six, and there are three total possible damaged cells. In general,

$$Q_A(1) = (N - 1)N \text{ and } Q_B(1) = N. \quad (14)$$

For the next shot (m = 2), the probability of nonperforation for the specific case of N = 3 is given by

$$P(2) = 3*2/3^2 + 3/3^2 = 1. \tag{15}$$

In general,

$$P(2) = (N-1)N/N^2 + N/N^2 = 1. \tag{16}$$

The second shot will produce the possibilities shown in Figure 6.

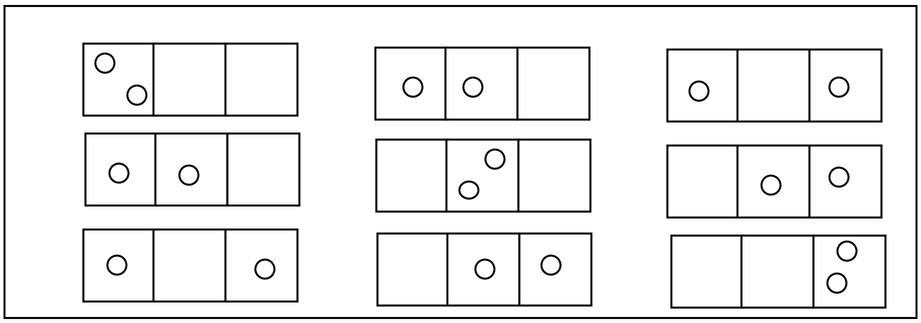

Figure 6. Possible outcomes from the second impact for N = 3.

The number of undamaged cells shown in Figure 6 is 12, and the number that have been impacted once is also 12. Thus, $Q_A(2) = 12$, $Q_B(2) = 12$, and

$$P(3) = 12/3^3 + 12/\,3^3 = 8/9 \tag{17}$$

for N = 3.

In general, the third shot will impact a target represented by either (N – 1, 0, 1) or (N – 2, 2, 0). There will be N combinations where a single cell has been hit twice. All other combinations will have two cells with one impact and N – 2 undamaged cells. There are N(N – 1) of these combinations (N > 1). Thus,

$$Q_A(2) = (N-1)N + (N-2)N(N-1), \text{ and } Q_B(2) = 2N(N-1). \tag{18}$$

This gives

$$\begin{aligned} P(3) &= ((N-1)N + (N-2)N(N-1))/N^3 + ((2N(N-1))/N^3 \\ &= ((N)^3 - N)/(N)^3 = 1 - 1/N^2\,. \end{aligned} \tag{19}$$

To see the result for N = 3, note that there are six different arrangements resulting from the second impact. Symmetry arguments can be used to reduce this number to three. Figure 7 shows, in effect, one-third of the outcomes for the third shot. All configurations are not represented in this figure. For instance, the case where the array has three impacts in the center cell is not shown. However, the notation for this outcome (N – 1, 0, 0, 1) is the same as that of

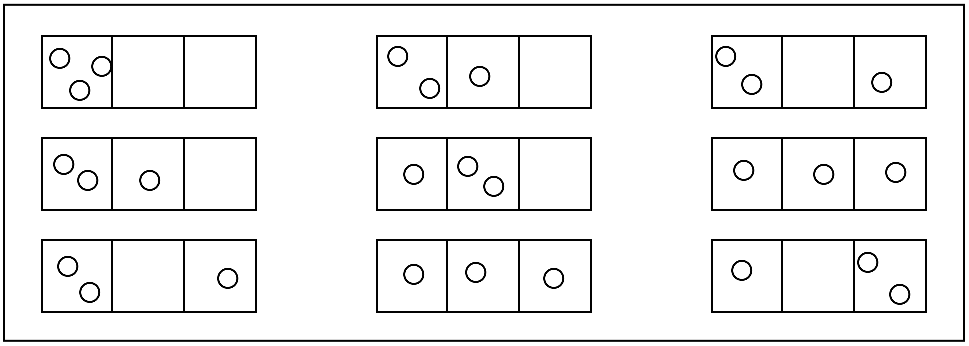

Figure 7. One-third of the possible outcomes from the third impact for N = 3 (not all configurations are represented in the figure).

the array in the first column and first row in Figure 7. One cell in the top left array in Figure 7 has been perforated, so that $P(3) = 8/9$ for $N = 3$.

The same line of reasoning can be applied for the fourth shot. After the third shot, there will be N combinations $(N - 1, 0, 0, 1)$, where a single cell has been hit three times. This set of combinations will no longer be considered in calculating Q_A and Q_B, since it has been perforated. A second combination $(N - 2, 1, 1, 0)$ will be one that has a single hit and a double hit. This can happen if a shot impacts an undamaged cell in an array that has two hits in one box or the shot impacts an array in a cell that has already been hit once. The number of ways where this can happen for these two cases is $N(N - 1)$ and $2N(N - 1)$, respectively. The third arrangement will be $(N - 3, 3, 0, 0)$ for arrays with three or more cells. Its frequency is $N(N - 1)(N - 2)$. Multiplying the number of cells with no impacts in all unperforated arrays times the frequency of occurrence gives

$$Q_A(3) = (N(N-1) + 2N(N-1))(N-2) + N(N-1)(N-2)(N-3). \tag{20}$$

Similarly for one-impact cell arrays,

$$Q_B(3) = (N(N-1) + 2N(N-1)) + 3N(N-1)(N-2). \tag{21}$$

Grouping terms and simplifying,

$$P(4) = 1 - 4/N^2 + 3/N^3 . \tag{22}$$

By similar reasoning, it can be shown that

$$P(5) = 1 - 10/N^2 + 15/N^3 - 6/N^4 , \tag{23}$$

$$P(6) = 1 - 20/N^2 + 45/N^3 - 26/N^4 , \tag{24}$$

and

$$P(7) = 1 - 35/N^2 + 105/N^3 - 56/N^4 - 105/N^5 + 90/N^6 . \qquad (25)$$

Note that P(3) = P(4) = P(5) = P(6) = P(7) = 0 for N = 1, P(5) = P(6) = P(7) = 0 for N = 2, and P(7) = 0 for N = 3.

A general solution with a somewhat different approach and using a combinatorial notation has been developed and is presented in an appendix to this report (Segletes, 2003). Using this solution, the probabilities of nonperforation for larger values of impacts can be calculated, although the algebra begins to be quite lengthy as the number of impacts increases. There is no good compact solution for the general case (Tanenbaum, 2003).

It is straightforward to show how these velocity effects would affect the value of P(m), the probability of nonperforation through the m^{th} shot. Take a patterned armor with a square cell edge length of 100 mm. It has a zone of low performance at the cell edge of 2 mm. Assume the limit velocity of the cell is 800 m/s against the 12.7-mm AP-M8 machine gun round. Assume the zone of low performance goes away at a striking velocity of 700 m/s and the cell can defeat two impacts at a velocity of 600 m/s. Let the dispersion of the machine gun be 1 mil and use the relation between A and the standard deviation of impacts from the aim point as derived in de Rosset and Wald (2002).

Figure 8 shows how P(7) varies as a function of range. There are steep increases in performance as a function of range. This steepness results from the simplification that the performance changes at a specific velocity. In reality, the transition would be smoother.

From 0 to 223 m, P(7) is 0 because the striking velocity is above the limit velocity of the armor for this particular threat (12.7-mm machine gun). From 223 to 476 m, the armor is able to stop single impacts on a cell but cannot stop them if they hit the zone of poor performance. From 476 to 754 m, the zone of poor performance goes away. Above 754 m, a single cell is able to defeat two impacts. Note that there is not much change in P(7) in going from a single-impact cell capability to a double-impact cell capability. It may be that, in many cases, the capability of the armor to stop multiple impacts at long ranges is so high that there is little room for improvement.

Consider now the trade-off between an armor cell that has a high limit velocity but single-impact capability vs. an armor cell that has a lower limit velocity but double-impact capability. Let the high limit velocity be 900 m/s vs. the 12.7-mm machine gun and let the lower impact velocity cell be 800 m/s against the same threat. Both armors are 100 mm on a side, and neither has a zone of low performance near the seam. Figure 9 shows P(7) as a function of range for each of these patterned armors.

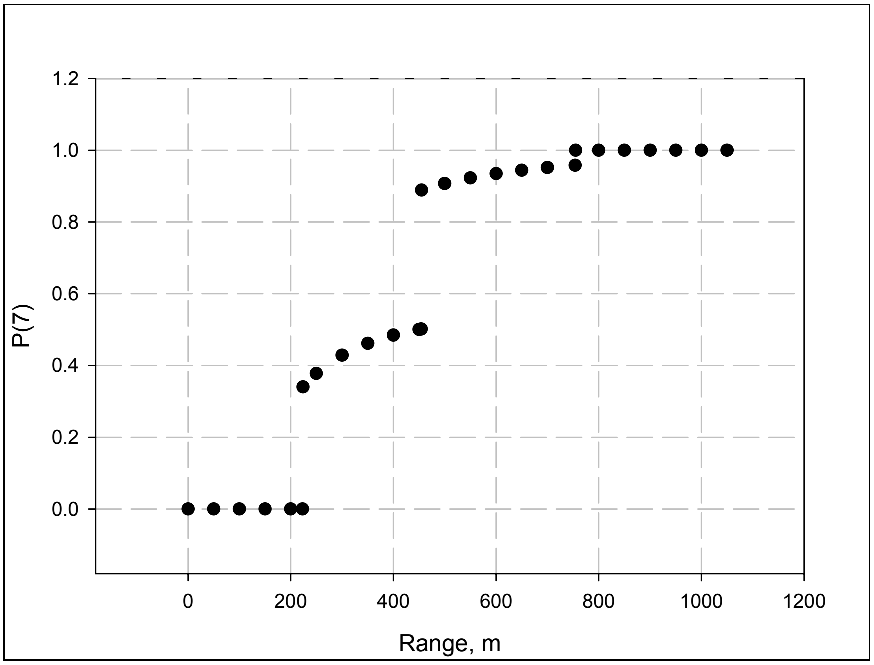

Figure 8. P(7) as a function of range, indicating velocity effects.

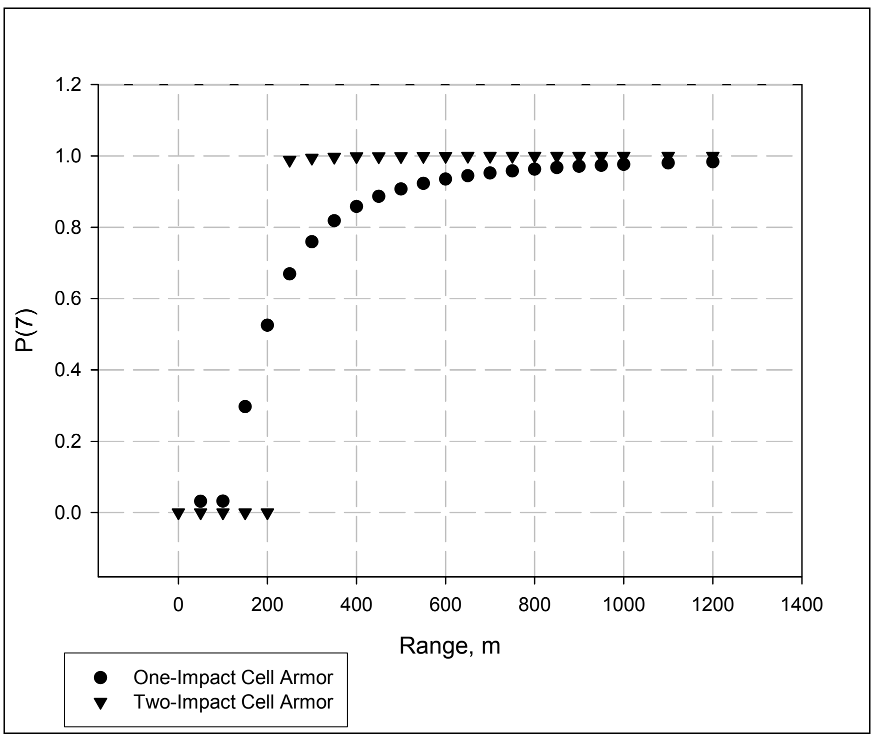

Figure 9. Performance comparison between a one-impact cell and a two-impact cell armor.

At short ranges, the one-impact cell outperforms the two-impact cell; but above the 223-m range, the two-impact cell has a higher value of P(7). The value of having a high-performance (i.e., high limit velocity) armor is negated (to some extent) at very close range due to the multiple impacts hitting a small area.

It is clear that there are many different types of comparisons that can be made between patterned armors with various performance characteristics. However, unless there is a well-posed performance criterion, the methodology presented here is of little value. Being well-posed means that the performance criterion recognizes the nature of automatic weapons fire. For instance, a well-posed criterion might be stated as the requirement of the armor to defeat a seven-round burst of a 12.7-mm machine gun at a 500-m range, with a probability of more than 95%. The methodology presented here could then be used to ascertain whether the armor met the criterion. In the example presented in Figure 9, the two-impact cell armor meets the requirement while the one-impact cell armor does not.

As an added note, the methodology presented here might also be used for monolithic armor if it is found that an impact affects an area adjacent to it where a second impact is not defeated. The cell size would be associated with the dimensions of the affected area, but there would be no zone of low performance.

4. Summary

The goal of the armor designer has always been to defeat any and all threats. While this is a worthy goal, it must be recognized that light armor attacked by automatic weapons fire will always be defeated under some circumstances. Consequently, realistic performance criteria need to be decided upon.

Characteristics of an ideal patterned armor were presented. The types of data needed to determine performance were discussed. These included such items as the ballistic limit for a given cell, the variation in performance as a function of hit location on the cell, and the effect of an impact on a neighboring cell. These data could then be used in conjunction with the presented methodology to determine if the patterned armor met a well-posed multiple-hit criterion. The methodology presented here could also be used to conduct trade-off studies between competing armor technologies or examine the effect of parameter variations (cell size, limit velocity of a cell, individual cell capability to withstand multiple impacts, etc.).

5. References

de Rosset, W. S.; Wald, J. K. *Analysis of Multiple-Hit Criterion for Ceramic Armor;* ARL-TR-2861; U.S. Army Research Laboratory: Aberdeen Proving Ground, MD, September 2002.

de Rosset, W. S. *Reactive Armor Model Sensitivity Studies*; ARL-TR-1849; U.S. Army Research Laboratory: Aberdeen Proving Ground, MD, November 1998.

Lambert, J. P.; Jonas, G. H. *Towards Standardization in Terminal Ballistics Testing: Velocity Representation*; BRL Report 1852, U.S. Army Ballistic Research Laboratory: Aberdeen Proving Ground, MD, January 1976.

Mascianica, F. S. *Ballistic Technology of Lightweight Armors*; AMMRC TR 76-15; U.S. Army Materials and Mechanical Research Center: Watertown, MA, May 1976.

Segletes, S. B. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD. Private communication, May 2003.

Tanenbaum, P. J. U.S. Army Research Laboratory, Aberdeen Proving Ground, MD. Private communication, May 2003.

Appendix: General Formulation for the Probability of Nonperforation of a Patterned Armor With Cells Capable of Withstanding More Than One Impact

Combinatorial Problem for *m* Hits on an *N*-tile Target Array

Before presenting the general form of the method, let us first consider a specific problem that will help identify the basic strategy for solution. Consider the problem of seven random impacts on a four-tile[1] target array, where three hits upon any given tile are required in order to defeat the tile and thus the array.

Let us name the tiles “a,” “b,” “c,” and “d.” With a problem of this limited scale, one may deduce from inspection that the only combinations for surviving seven hits on four tiles are of the form “aabbccd,” where one of the tiles (in this case, “d”) is struck once and all the other tiles (“a,” “b,” and “c”) are each struck twice. The two hits on tile “a” may occur at the following possible positions among the seven-hit impact sequence:

aa▪▪▪▪▪	▪aa▪▪▪▪	▪▪aa▪▪▪	▪▪▪aa▪▪	▪▪▪▪aa▪	▪▪▪▪▪aa
a▪a▪▪▪▪	▪a▪a▪▪▪	▪▪a▪a▪▪	▪▪▪a▪a▪	▪▪▪▪a▪a	
a▪▪a▪▪▪	▪a▪▪a▪▪	▪▪a▪▪a▪	▪▪▪a▪▪a		
a▪▪▪a▪▪	▪a▪▪▪a▪	▪▪a▪▪▪a			
a▪▪▪▪a▪	▪a▪▪▪▪a				
a▪▪▪▪▪a					

where the “•” denotes an as yet undesignated impact upon a tile other than “a.” This yields 21 possibilities, which also equals C (7, 2) = 7!/(5! 2!) = 21, signifying the number of two-element subsets from a seven-element set. *For each* of these 21 possibilities, the question is then how could the two impacts upon tile “b” be arranged from the remaining five undesignated impacts. The two element subsets for tile “b” from the five remaining undesignated impacts are as follows:

bb▪▪▪	▪bb▪▪	▪▪bb▪	▪▪▪bb
b▪b▪▪	▪b▪b▪	▪▪b▪b	
b▪▪b▪	▪b▪▪b		
b▪▪▪b			

[1] In this appendix, a tile is equivalent to a cell.

This yields 10 possibilities equaling C (5, 2), the number of two-element subsets from a five-element set. Because each of these impact sequences upon tile "b" applies to the undesignated shots *for each* of the two-element subsets for impacts on tile "a," the total ways to order two impacts on "a" *and* two impacts on "b" among the seven-shot sequence produces 21 × 10 = 210 possibilities. Of the remaining three undesignated impacts, the ways to choose two impacts on tile "c" are as follows:

cc ▪ ▪ cc
c ▪ c

This yields three possibilities equaling C (3, 2). The number of ways, therefore, to order two impacts on "a" *and* two impacts on "b" *and* two impacts on "c" among the seven-shot sequence produces 210 × 3 = 630 possibilities. There is only one way to choose from among the lone remaining undesignated impact equaling C (1, 1) = 1, which must be upon tile "d." And so there remain 630 × 1 = 630 possibilities to order two impacts on tile "a" *and* two impacts on "b" *and* two impacts on "c" *and* one impact on "d."

Finally, the combinations examined were for the assumed case of tile "d" receiving only one impact. In fact, there are four possibilities to choose from as to which one tile gets struck only once. So, having solved the problem for one hit on tile "d," multiplying the result by C(4, 1) = 4 will cover the chances that it was, in fact, tiles "a," "b," or "c" being hit only once. So the possible orderings of survivable impacts will number 630 × 4 = 2520 possible ways of the seven impacts being spread 2 + 2 + 2 + 1 upon the four tiles.

The number of ways that seven distinct shots can be distributed randomly among the four target tiles is simply $4^7 = 16384$, since for each of the seven successive impacts, there are four possible outcomes as to which tile is struck. The survivability rate for this seven-shot sequence on the four-tile target array, assuming that all shots are independently and uniformly distributed, is therefore $P = 2520/16384 = 0.1538$.

In this example, we explicitly counted the array-survival orderings because the combinations of ways to survive were so limited–namely limited to a spread of 2 + 2 + 2 + 1 impacts upon the elements of the four-tile array. It the number of tiles were to be increased, the orderings of ways that seven impacts could produce a surviving target array would grow quite rapidly, quickly outstripping the number of impact-sequence orderings resulting in a defeat of the array. And while it is largely a matter of preference, the general method will be developed to explicitly count not the number of surviving impact sequences, but rather the number of defeating impact sequences. The survival probability is, at that point, simply calculated by subtracting the probability of defeat from unity.

The example shown, however, does not highlight the one problem that arises when tabulating impact sequences for cases when the array is randomly struck by a large number of impacts. For the situation of counting impact sequences that cause array defeat, the following applies: when

there exists a sequence of m impacts in which two (or more) of the N tiles can be individually defeated, the method of totaling the orderings for the array, in which the total number of tiles multiplies the number of "defeat" orderings that a single tile contributes, must be subjected to a correction. A subtractive correction is necessary because a single impact sequence that results in, for example, two tiles being defeated is overcounted with the basic method, since the sequence shows up in the "defeated" tabulations for both tiles "a" and "b," even though the twice-counted defeat arose from only a single impact sequence. Unfortunately, the correction itself requires a subtractive correction, when the possibility arises that there are three tiles that could have been simultaneously defeated by the impact sequence. Since two successive subtractive corrections make for an addition, successive corrections will alternate in sign. This alternating correction might need to be successively applied up to and until the limiting case of all N tiles being defeated by the m impacts, should m be large enough to do so. In the field of combinatorics, this approach to tabulating the impact sequences is known, as an inclusion/exclusion approach.

Note that an analogous correction is required when counting impact sequences that do not result in array defeat, and so there is no trivial way around this problem. It, likewise, must rely on an inclusion/exclusion approach, after a similar fashion.

We now present the general result to the problem of calculating the likelihood that an N-tile target array will survive an m-shot sequence if k shots are required upon any single tile to defeat it and thus the array. First, we lay out the following explicit definitions:

- P – probability that a target array will survive a random volley of impacts (i.e., hits).
- N – number of (identical) target tiles in the target array.
- m – number of hits randomly spread upon the target array.
- k – threshold number of hits on a given tile required to defeat that tile (and thus the array).
- j_n – index denoting number of hits upon the n'th defeated tile arising from the m-shot sequence.
- $\binom{n}{k}$ – the binomial coefficient that counts the number of ways one can assign each member of an n-set with either of two labels—k of them getting labeled *chosen* and the remaining $n-k$ getting labeled *not chosen*. The binomial coefficient represents the number of k-element subsets from an n-element set, is equal to $\frac{n!}{(n-k)!k!}$, and is alternately represented as $C(n, k)$. These are the familiar elements of Pascal's triangle.

- $\binom{n}{k_1, k_2, \ldots, k_t}$ – the multinomial coefficient, a generalization of the binomial coefficient, that counts the number of ways one can assign each member of an n-set one of t labels. The label indices k_1, ..., k_t are nonnegative integers, and $k_1 + \cdots + k_t = n$. The multinomial coefficient is equal to $\binom{n}{k_1, k_2, \ldots, k_t} = \binom{n}{k_1}\binom{n-k_1}{k_2}\binom{n-(k_1+k_2)}{k_3}\cdots\binom{n-(k_1+\cdots+k_{t-1})}{k_t} = \frac{n!}{k_1!k_2!\cdots k_t!}$.

The form that follows arises from explicitly counting the m-shot sequences that defeat the array, dividing them by the total possible number of m-shot sequences on an array of N target tiles (N^m), and subtracting that probability from 1 to get the probability of surviving a sequence of m shots at the array. The following formula would seem to apply (though it has not been rigorously proven):

$$P = 1 - \frac{1}{N^m}\left\{\binom{N}{1}\sum_{j_1=k}^{m}\binom{m}{j_1}(N-1)^{m-j_1} - \binom{N}{2}\sum_{j_1=k}^{m-k}\sum_{j_2=k}^{m-j_1}\binom{m}{j_1, j_2, m-j_1-j_2}(N-2)^{m-j_1-j_2} \right.$$
$$\left. + \binom{N}{3}\sum_{j_1=k}^{m-2k}\sum_{j_2=k}^{m-j_1-k}\sum_{j_3=k}^{m-j_1-j_2}\binom{m}{j_1, j_2, j_3, m-j_1-j_2-j_3}(N-3)^{m-j_1-j_2-j_3} - \ldots\right\}. \qquad \text{(A-1)}$$

The first summation in the braces counts out the number of impact sequences for which a given tile is defeated (impacted k, $k + 1$ …, or m times) and multiplies the result by the number of tiles, i.e., C(N, 1). Beyond the first summation term, the finite sequence of subsequent summations represent correction terms needed to subtract out duplicately counted failures and/or corrections when more than one tile has been subjected to the lethal threshold of hits. The second summation grouping represents the correction for those limited cases when exactly two tiles in the array are defeated from the m-shot sequence. The third summation sequence corrects the correction for those cases when exactly three tiles are defeated, etc.

In the second term, for example, $\binom{N}{2}$ denotes the number of possible pairs of (defeated) tiles from an array of N tiles, and $\sum_{j_1=k}^{m-k}$ denotes a summation over the possible values for the number of impacts on the first tile of the pair. It shows that the first defeated tile must have at least k impacts. An upper limit is $m - k$ impacts, since second defeated tile must have at least k impacts as well. $\sum_{j_2=k}^{m-j_1}$ denotes a summation over the possible values for the number of impacts on the second tile of the pair. It shows that the second defeated tile must have at least k impacts but is bounded by $m - j_1$ impacts since there are only m impacts in total j_1, which are on the first defeated tile, $\binom{m}{j_1, j_2, m-j_1-j_2}$ is the multinomial coefficient representing $\binom{m}{j_1}\binom{m-j_1}{j_2}$. In this product, $\binom{m}{j_1}$ denotes the number of ways that j_1 impacts on the first defeated tile may be ordered among the m impacts, and $\binom{m-j_1}{j_2}$ denotes the number of ways that j_2 impacts on the second defeated tile

may be arranged among the remaining $m - j_1$ undesignated impacts. $(N-2)^{m-j_1-j_2}$ denotes the number of ways that the $m - j_1 - j_2$ undesignated nonlethal impacts may be ordered among the remaining $N - 2$ nondefeated tiles.

There are exactly N summation-term groupings in the solution, with leading multipliers C(N, 1), C(N, 2)…C(N, N), since a maximum of N tiles may simultaneously fail under any condition. In practical application, however, the maximum number of summation-term groupings is $\lfloor m/k \rfloor$ (i.e., in this case, the "floor" operator indicates the largest integer not greater than m/k) since this is the actual maximum number of tiles that may simultaneously fail from firing m shots onto tiles with a lethal threshold of k shots. The solution accounts for this since terms beyond such a value of m/k will have the leading summation of the term performed zero times. Thus, the number of summation-term groupings actually required is min $[N, \lfloor m/k \rfloor]$, depending on whether the lethal impact sequences are limited by the overall number of tiles (N) in, or the number of actual shots (m) on, the target array.

And while the summation-term groupings in the general solution may seem foreboding, the actual application of the equation to a quantifiable scenario is much less so. Take, for example, the case of an eight-hit distribution upon a 10-tile target, where three hits are required to defeat any given tile and thus the following array:

$$P = 1 - \frac{1}{10^8}\left\{10\left[56 \cdot 9^5 + 70 \cdot 9^4 + 56 \cdot 9^3 + 28 \cdot 9^2 + 8 \cdot 9 + 1\right] - 45\left[56\left(10 \cdot 8^2 + 5 \cdot 8 + 1\right)\right]\right\} \\ + 70\left(4 \cdot 8 + 1\right) + 56 = 0.637. \tag{A-2}$$

Similarly, if those same eight impacts had been distributed over three tiles, instead of ten, at least one of the tiles would necessarily have received three or more impacts, rendering that tile and thus the array defeated. Such a response is accurately captured by the following formula:

$$P = 1 - \frac{1}{3^8}\left\{3\left[56 \cdot 2^5 + 70 \cdot 2^4 + 56 \cdot 2^3 + 28 \cdot 2^2 + 8 \cdot 2 + 1\right] - 3\left[56\left(10 \cdot 1^2 + 5 \cdot 1 + 1\right)\right]\right\} \\ + 70\left(4 \cdot 1 + 1\right) + 56 = 0. \tag{A-3}$$

INTENTIONALLY LEFT BLANK.

NO. OF COPIES	ORGANIZATION
2	DEFENSE TECHNICAL INFORMATION CENTER DTIC OCA 8725 JOHN J KINGMAN RD STE 0944 FT BELVOIR VA 22060-6218
1	COMMANDING GENERAL US ARMY MATERIEL CMD AMCRDA TF 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	INST FOR ADVNCD TCHNLGY THE UNIV OF TEXAS AT AUSTIN 3925 W BRAKER LN STE 400 AUSTIN TX 78759-5316
1	US MILITARY ACADEMY MATH SCI CTR EXCELLENCE MADN MATH THAYER HALL WEST POINT NY 10996-1786
1	DIRECTOR US ARMY RESEARCH LAB AMSRL D DR D SMITH 2800 POWDER MILL RD ADELPHI MD 20783-1197
1	DIRECTOR US ARMY RESEARCH LAB AMSRL CS IS R 2800 POWDER MILL RD ADELPHI MD 20783-1197
3	DIRECTOR US ARMY RESEARCH LAB AMSRL CI OK TL 2800 POWDER MILL RD ADELPHI MD 20783-1197
3	DIRECTOR US ARMY RESEARCH LAB AMSRL CS IS T 2800 POWDER MILL RD ADELPHI MD 20783-1197

NO. OF COPIES	ORGANIZATION
	ABERDEEN PROVING GROUND
2	DIR USARL AMSRL CI LP (BLDG 305) AMSRL CI OK TP (BLDG 4600)

NO. OF COPIES	ORGANIZATION
1	DARPA L CHRISTODOULOU 3701 N FAIRFAX DR ARLINGTON VA 22203-1714
1	COMMANDANT US ARMY ARMOR CENTER ATZK CDP M BOSEMER FORT KNOX KY 40121
1	US ARMY TARDEC AMSTA TR R MS 263 D TEMPLETON 6501 E 11 MILE RD WARREN MI 48397-5000
3	COMMANDER US ARMY ARDEC S MUSSALI T SEBASTO J HEDDERICH PICATINNY ARSENAL NJ 07806-5000
2	COMMANDER US ARMY AMRDEC D LOVELACE M SCHEXNAYDER REDSTONE ARSENAL AL 35898-5250

ABERDEEN PROVING GROUND

NO. OF COPIES	ORGANIZATION
53	DIR USARL AMSRL WM K BOYD B BURNS J SMITH E SCHMIDT T WRIGHT AMSRL WM B A HORST T KOGLER AMSRL WM M B FINK AMSRL WM MB B DOOLEY C HOPPEL E RIGAS B SPURGEON AMSRL WM MC E CHIN J LASALVIA J SWAB AMSRL WM MD V CHAMPAGNE W DEROSSET (20 CPS) B DOWDING S GRENDAHL D HELFRITCH W ROY D SNOHA F STENTON R WOODMAN AMSRL WM T T HAVEL M ZOLTOSKI AMSRL WM TA B BRUCHEY M BURKINS B GOOCH T HADUCH W ROWE J RUNYEON AMSRL WM TC R COATES AMSRL WM TD S SEGLETES